The magnetohydrodynamic squeeze film

By DENNIS C. KUZMA, E. ROLAND MAKI
Research Laboratories, General Motors Corporation, Warren, Michigan
AND RUSSELL J. DONNELLY
University of Chicago, Chicago, Illinois

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Magnetohydrodynamic squeeze films are investigated theoretically and experimentally. The theory of magnetohydrodynamic lubrication as applied to squeeze films is extended to include fluid-inertia effects and buoyant forces. Excellent agreement is obtained between theory and experiment.

1. Introduction

There has been considerable interest in recent years in magnetohydrodynamic lubrication theory. A number of analytical studies of magnetohydrodynamic bearings have been published (e.g. Snyder 1962; Hughes & Elco 1962*a, b*; Hughes 1963; Kuzma 1963*a, b*), but we are not aware of any experimental verification of these studies. The object of this paper is to present both the theory and experimental results for a hydromagnetic squeeze film using mercury between two circular flat plates. The magnetic field is applied perpendicular to the two plates, and the separation of the plates is measured as a function of time. The conditions of the experiment are not the same as those assumed in the calculations of Kuzma (1963*b*), and his analysis is extended to include the effects of fluid inertia. Buoyant forces acting on the plate are also included in the analysis. The results of the experiment are in excellent agreement with the theory.

2. Apparatus

A simplified diagram of the apparatus is shown in figure 1, and a detailed photograph is shown in figure 2, plate 1. The squeeze film is generated between two circular glass plates which are 8·5 in. in diameter. Each plate has a convexity of approximately 70 μin. with the highest point located near the centre of the plates. The effective area of the top plate (for buoyancy considerations) is $A = 65{\cdot}1\,\text{in.}^2$ and the area of the container $A_c = 78{\cdot}5\,\text{in.}^2$. The plates are maintained parallel by using a pair of flexures. Three strain beams are equally spaced around the upper plate to determine its position relative to the lower plate. Each strain beam is connected to a separate strip-chart recorder and then calibrated with respect to a dial indicator on the upper plate. The load on the upper plate consists of its own weight plus interchangeable dead weights placed on the upper surface. This load is counterbalanced by a simple lever which is tied with a monofilament line. The line is cut to start the experiment.

The magnet was that of the 32·5 in. cyclotron at the University of Chicago. This magnet has 8·75 in. spacing between poles, and the field is homogeneous to within 0·1 % in the region of the glass plates.

The mercury used in hydromagnetic experiments must be cleaned with great care. The procedures developed at the University of Chicago are as follows. The mercury is first cleaned by means of a mercury oxifier and gold filter to remove base metals. Next, the mercury is sprayed successively through towers of nitric acid, sodium hydroxide, distilled water, freon cleaning fluid, and finally alcohol. The mercury was then exposed to vacuum overnight and stored in clean glass flasks. The glass plates and other apparatus were cleaned thoroughly before assembly, and the mercury was added just before use. A continuous film of mercury was obtained between the plates before positioning in the magnet. The apparatus was rolled into the magnet on rails and was carefully levelled in position. A photograph of the apparatus in position between the pole pieces is shown in figure 3, plate 1.

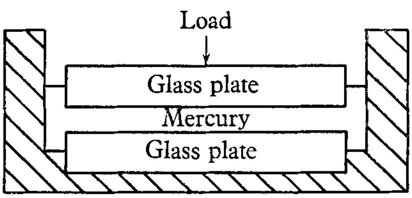

FIGURE 1. Simplified diagram of squeeze film apparatus.

3. Experimental results

Theoretical and experimental results are shown in figures 4 and 5. The experimental points were taken directly from the strip-chart records of the experiment. In both cases, the initial displacement of the plates (i.e. the thickness of the film of mercury) was $h_0 = 0{\cdot}073$ in. and the load was 27·51 lb. Buoyancy forces kept the plates separated at equilibrium. The equilibrium height is $h_\infty = 0{\cdot}025$ in. in figure 4 and 0·010 in. in figure 5. This equilibrium height is changed by varying the amount of mercury in the container. The temperature of the mercury was 23·5 °C at which the viscosity is 1·535 cP and the density is 13·54 g/cm^3. Two major sources of error are present. First, the pen records could be read only to an accuracy of $\pm 0{\cdot}0003$ in. Further, the analysis gives only a first-order approximation to fluid inertia terms. This error is greatest at low magnetic field strengths.

4. Analysis

The analysis for magnetohydrodynamic squeezing flow in the usual lubrication approximation has been given by Kuzma (1963*b*). However, in order to interpret the present experiment it is necessary to allow for the buoyant force on the partially submerged upper plate. Furthermore, since the Reynolds' numbers ($\mathscr{R} = |h\dot{h}/\nu|$) during the experiment achieve values as high as 20, it is clearly necessary to make allowance for fluid inertia terms. We shall do this by a simple

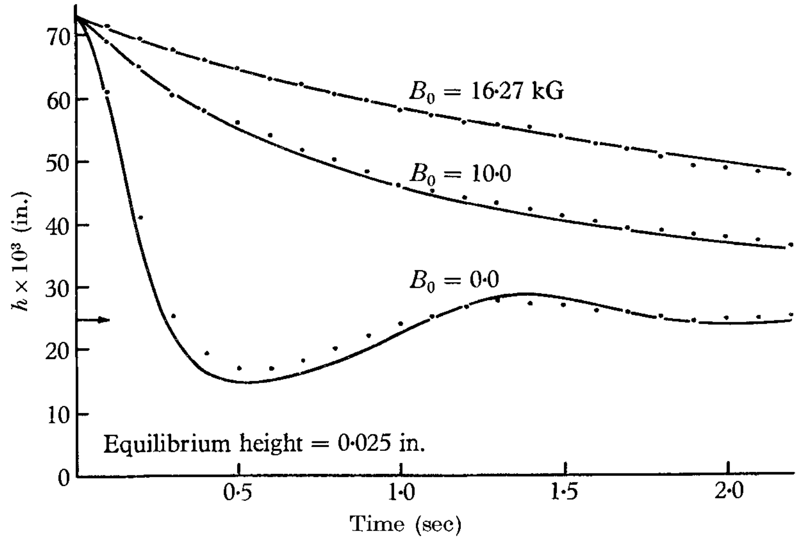

FIGURE 4. Height *vs* time for various magnetic field strengths and buoyant equilibrium height of 0·025 in. —, Theory; . . ., experiment.

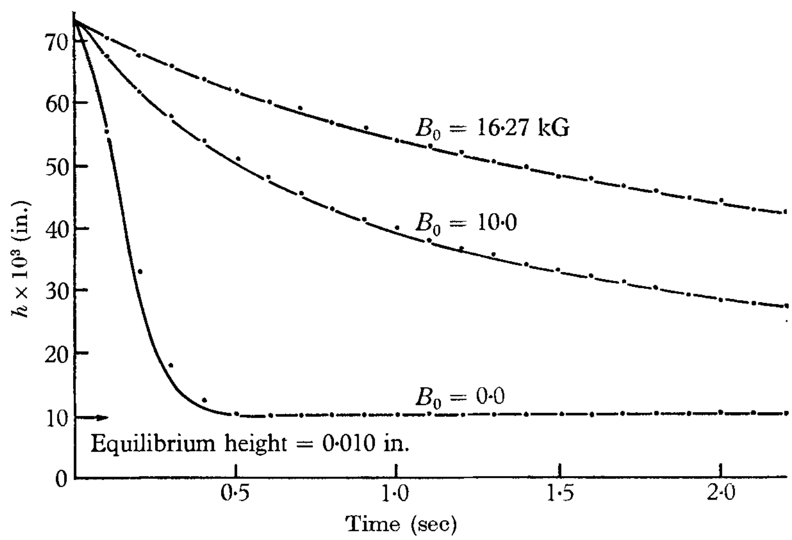

FIGURE 5. Height *vs* time for various magnetic field strengths and buoyant equilibrium height of 0·010 in. —, Theory; . . ., experiment.

extension of the approximation used by Jackson (1962). Referring to figure 1, the equation of motion of the upper plate is

$$\ddot{h}W/g = F_s + F_b, \tag{1}$$

where W is the load which includes the weight of the plate, F_s is the squeeze film force, and F_b is the buoyant force acting on the plate. The buoyant force may be determined by the use of a buoyant spring constant k:

$$F_b = k(h_\infty - h). \tag{2}$$

The buoyant spring constant may be determined from the geometry shown in figure 1. Thus

$$k = \rho g A A_c/(A_c - A), \tag{3}$$

where A is the effective area of the plate, A_c is the area of the container, and ρ is the density of the mercury.

The squeeze film force acting on the plate comes from an analysis of hydromagnetic squeezing flow. The equation of conservation of momentum in hydromagnetics is

$$\rho(\partial \mathbf{u}/\partial t) + \rho(\mathbf{u}.\nabla)\,\mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} + \sigma(\mathbf{u}\times\mathbf{B})\times\mathbf{B}. \tag{4}$$

If the applied magnetic field B_0 is in the z-direction, the momentum equation in the r-direction becomes

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} - \sigma B_0^2 u + \mu\left(\frac{\partial^2 u}{\partial y^2} - \frac{u}{r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right). \tag{5}$$

The above equation adds the electromagnetic force term to Jackson's (1962) momentum equation

Successive approximations $u^{(1)}, u^{(2)}, \ldots,$ are used to determine the velocity profile, given by

$$\mu\frac{\partial^2 u^{(i)}}{\partial y^2} - \sigma B_0^2 u^{(i)} = \frac{\partial p}{\partial r} + \rho u^{(i-1)}\frac{\partial u^{(i-1)}}{\partial r} + \rho\frac{\partial u^{(i-1)}}{\partial t} - \mu\left(\frac{\partial^2 u^{(i-1)}}{\partial r^2} + \frac{1}{r}\frac{\partial u^{(i-1)}}{\partial r} - \frac{u^{(i-1)}}{r^2}\right) \quad (i = 2, 3, \ldots). \tag{6}$$

The velocity distribution must satisfy the continuity equation

$$-\pi r^2 \dot{h} = \int_0^h 2\pi r u\, dy. \tag{7}$$

The velocity profile which neglects inertia terms (Kuzma 1963) will be used as a first approximation. This velocity is

$$u^{(1)} = -r\dot{h}\left(\frac{M_0}{h_0}\right)\frac{\cosh M_0 y/h_0 - 1 - \tanh M_0 h/2h_0 \sinh M_0 y/h_0}{4\tanh M_0 h/2h_0 - 2M_0 h/h_0}, \tag{8}$$

where h_0 is the initial film thickness and

$$M_0 = B_0 h_0 (\sigma/\mu)^{\frac{1}{2}} \tag{9}$$

is the initial Hartmann number. The velocity profile from equation (8) may be substituted into equation (6). This gives

$$\mu\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u = \frac{\partial p}{\partial r} - \rho r\ddot{h}\left(\frac{M_0}{h_0}\right) f(y) + \frac{\rho r\dot{h}^2}{2}\left(\frac{M_0}{h_0}\right)^2 \frac{\operatorname{sech}^2 M_0 h/2h_0 \sinh M_0 y/h_0}{4\tanh M_0 h/2h_0 - 2M_0 h/h_0} - \frac{\rho r\dot{h}^2}{2}\left(\frac{M_0}{h_0}\right)^2 \frac{f(y)\tanh^2 M_0 h/2h_0}{\tanh M_0 h/2h_0 - M_0 h/2h_0} + \rho r\dot{h}^2\left(\frac{M_0}{h_0}\right) f^2(y), \tag{10}$$

where
$$f(y) = \frac{\cosh M_0 y/h_0 - 1 - \tanh M_0 h/2h_0 \sinh M_0 y/h_0}{\tanh M_0 h/2h_0 - M_0 h/2h_0} \tag{11}$$

and $u^{(2)}$ has been replaced by u. Equation (10) may be solved for u, and the result substituted into equation (7). After integration and rearrangement, this equation becomes

$$\begin{aligned}\frac{\partial p}{\partial r} = &-\frac{\mu a^3 \dot{h} r}{4} \frac{1}{\tanh ah/2 - ah/2} \\ &+ \frac{\rho a^3 \ddot{h} r}{2}\left[\frac{3}{4a^2}\frac{1}{\tanh ah/2 - ah/2} + \frac{h}{8a}\frac{\tanh^2 ah/2}{(\tanh ah/2 - ah/2)^2}\right] \\ &+ \frac{\rho \dot{h}^2 a^3 r}{2(\tanh ah/2 - ah/2)^3}\left[-\frac{5h}{32} + \left(\frac{5}{16a} - \frac{ah^2}{32}\right)\tanh ah/2 \right. \\ &\left. + \frac{h}{32}\tanh^2 ah/2 + \left(\frac{1}{6a} + \frac{ah^2}{32}\right)\tanh^3 ah/2\right],\end{aligned} \tag{12}$$

where
$$a = M_0/h_0. \tag{13}$$

The boundary condition for the pressure is

$$p(R) = 0, \tag{14}$$

R being the radius of the plate. Equation (12) may be integrated to give

$$\begin{aligned}p = \frac{R^2 - r^2}{2}&\left\{\frac{\mu a^3 \dot{h}}{4}\frac{1}{\tanh ah/2 - ah/2}\right. \\ &+ \frac{\rho a^3 \ddot{h}}{2}\left[\frac{3}{4a^2}\frac{1}{\tanh ah/2 - ah/2} + \frac{h}{8a}\frac{\tanh^2 ah/2}{(\tanh ah/2 - ah/2)^2}\right] \\ &+ \frac{\rho \dot{h}^2}{2}\frac{a^3}{(\tanh ah/2 - ah/2)^3}\left[-\frac{5h}{32} + \left(\frac{5}{16a} - \frac{ah^2}{32}\right)\tanh ah/2\right. \\ &\left.\left. + \frac{h}{32}\tanh^2 ah/2 + \left(\frac{1}{6a} + \frac{ah^2}{32}\right)\tanh^3 ah/2\right]\right\}.\end{aligned} \tag{15}$$

The squeeze film force acting on the plate then becomes

$$F_s = \int_0^R 2\pi r p \, dr \tag{16}$$

or
$$\begin{aligned}F_s = \frac{\pi R^4}{4}&\left\{\frac{\mu \dot{h}}{4}\frac{1}{\tanh ah/2 - ah/2}\right. \\ &+ \frac{\rho \ddot{h}}{2}\left[\frac{3a}{4}\frac{1}{\tanh ah/2 - ah/2} + \frac{a^2 h}{8}\frac{\tanh^2 ah/2}{(\tanh ah/2 - ah/2)^2}\right] \\ &+ \frac{\rho \dot{h}^2}{2}\frac{a^3}{(\tanh ah/2 - ah/2)^3}\left[-\frac{5h}{32} + \left(\frac{5}{16a} - \frac{ah^2}{32}\right)\tanh ah/2\right. \\ &\left.\left. + \frac{h}{32}\tanh^2 ah/2 + \left(\frac{1}{6a} + \frac{ah^2}{32}\right)\tanh^3 ah/2\right]\right\}.\end{aligned} \tag{17}$$

Equations (17) and (2) may be substituted into equation (1) and rearranged to give

$$\left[\frac{W}{g}-\frac{\pi R^4}{4}\left(\frac{3a\rho}{8}\frac{1}{\tanh ah/2-ah/2}+\frac{\rho a^2 h}{16}\frac{\tanh^2 ah/2}{(\tanh ah/2-ah/2)^2}\right)\right]\ddot{h}$$
$$=\frac{\pi R^4}{4}\left[\frac{\mu\dot{h}}{4}\frac{a^3}{\tanh ah/2-ah/2}\right]+k(h_\infty-h)$$
$$+\frac{\pi R^4}{8}\frac{\rho a^3\dot{h}^2}{(\tanh ah/2-ah/2)^3}\left[-\frac{5h}{32}+\left(\frac{5}{16a}-\frac{ah^2}{32}\right)\tanh ah/2\right.$$
$$\left.+\frac{h}{32}\tanh^2 ah/2+\left(\frac{1}{6a}+\frac{ah^2}{32}\right)\tanh^3 ah/2\right]. \quad (18)$$

The height may be determined as a function of time by solving equation (18) numerically, according to

$$h(i)=h(i-1)+\dot{h}(i-1)\,\Delta t+\tfrac{1}{2}\ddot{h}(i-1)\,(\Delta t)^2, \quad (19)$$
$$\dot{h}(i)=\dot{h}(i-1)+\ddot{h}(i-1)\,\Delta t. \quad (20)$$

The value for $\ddot{h}(i)$ may be found by substituting $h(i)$ and $\dot{h}(i)$ into equation (18). This calculation was done on a high-speed digital computer using a time increment of 0·001 sec.

5. Comments

The use of only one iteration seems to give excellent results. Furthermore, the presence of a magnetic field increases the accuracy of the iteration procedure since the inertia terms then become small compared to the combined viscous and inertia terms. At large values of the Hartmann number, the magnetic effects completely overshadow both the viscous and inertia effects.

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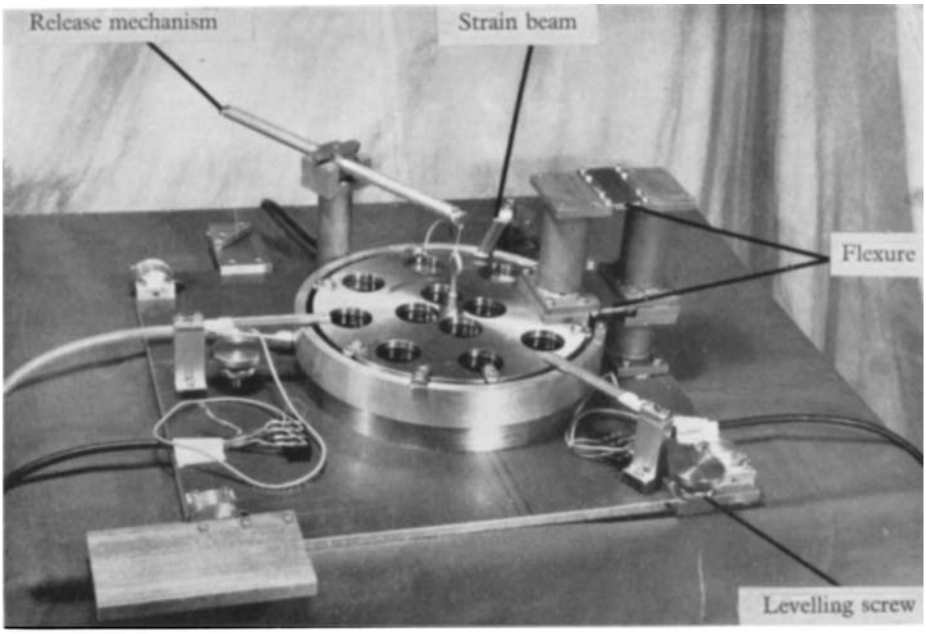

FIGURE 2. Experimental apparatus.

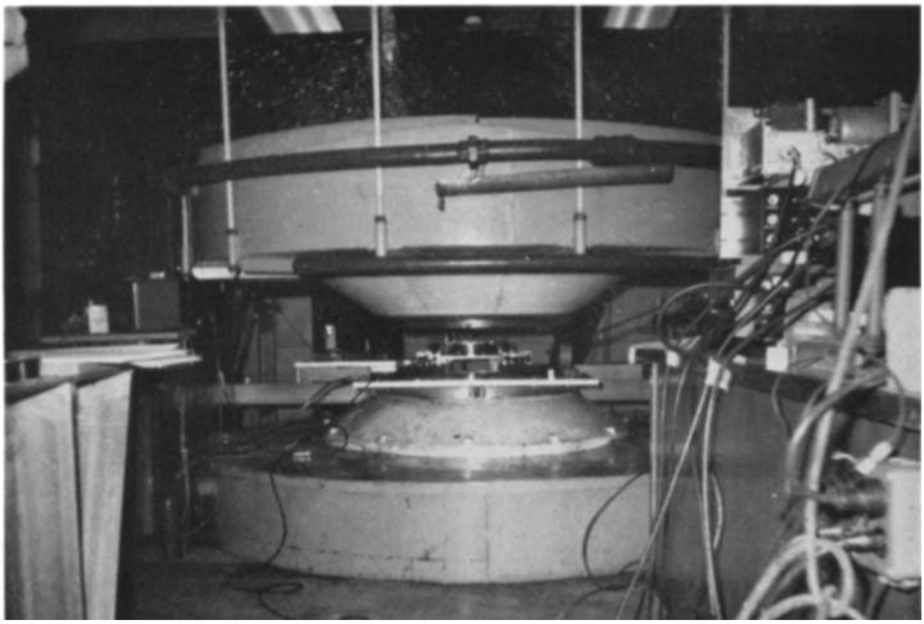

FIGURE 3. Apparatus in position between magnetic poles.